Data-Based and Statistical Reasoning

Measures of Central Tendency

- Mean: the average of the data. Is not outlier resistant
- Median: midpoint of data. If even number of data points, then median will be the average of two points. Outlier resistant.
    - If mean and median far from each other, indicates presence of outliers or skewed distribution.
- Mode: number that appears the most often in a set of data.

Distributions

- Normal Distribution: can transform any normal distribution to a **standard distribution** with a mean of zero and a standard deviation one 1.
- Skewed Distribution: Contains a **tail** on one side of the data set and is thus not symmetric.
    - Negative-Skewed: has a tail on the left, mean will be lower than the median
    - Positively-skewed: has a tail on the right, mean will be larger than the median.
- Bimodal Distribution: Has two peaks, can sometimes be measured as two different distributions.

Measures of Distribution

- Range: difference between the largest and smallest values of a data set. Heavily affected by presence of data outliers. Standard deviation can be approximated as ¼ * range
- Interquartile Range: The third quartile minus the first quartile
    - Quartiles: divide data into groups that comprise one-fourth of the entire data set.
        - To calculate position of first quartile: sort data in ascending order and multiply *n* by 1/4
        - If this is a whole number, the quartile is the mean of the value at this position and the next highest position
        - If this is a decimal, round up to the next whole number and take that as the quartile position.
        - For 3rd quartile, multiply n by 3/4. Do same process as first quartile.
    - Outliers are those points that fall outside of 1.5*IQR
- Standard Deviation: $\sigma = \sqrt{\sum \frac{(x_i - \bar{x})^2}{n-1}}$
    - If data point falls more than three standard deviations from the mean, it is considered an outlier.
    - On a normal distribution: 68-95-99 rule applies.
- Outliers: usually results from one of three causes:
    - True statistical anomaly
    - A measurement error
    - Distribution is not approximated by a normal distribution.

Probability

- Mutually Exclusive Outcomes: cannot occur at the same time
- Exhaustive set of outcomes: no other possible outcomes.

Calculations

- For independent events, probability of two or more events occurring at the same time is the product of their probabilities alone
- The probability of at least one of two events occurring is equal to the sum of their initial probabilities minus the probability that will both occur.

Addition rule : $P(A \cup B) = P(A) + P(B) - P(A \cap B)$

If A and B are mutually exclusive: $P(A \cup B) = P(A) + P(B)$

Multiplication rule : $P(A \cap B) = P(A) * P(B \mid A)$ or $P(B) * P(A \mid B)$

If A and B are independent: $P(A \cap B) = P(A) * P(B)$

Statistical Testing

Hypothesis Testing

- Null Hypothesis: hypothesis of equivalence, says that two populations are equal.
- Alternative Hypothesis: non-direction (not equal) or direction (greater than or less than)
- Z-tests or t-tests are commonly used tests. **Test Statistic** is calculated form collected data, and compared to a table in order to determine the likelihood that the statistic was obtained by random choice. This likelihood is known as the **p-value**.
- If p-value > **level of significance** (usually 0.05) then the null hypothesis cannot be rejected.
    - When null is rejected, results are statistically significant since there is a difference between the two groups.
    - Level of significance is the level of risk that is accepted for incorrectly rejecting the null hypothesis. Also known as a **type I error**.
        - Type I Error: Likelihood that we report a difference between the two population when one does not actually exist
    - Type II Error: incorrectly fail to reject the null hypothesis. When no difference is reported when there actually is one. (β)
    - Power: the probability of correctly rejecting the null hypothesis: 1-β
    - Confidence: the probability of correctly failing to reject the null hypothesis when no difference exists.

		Truth About the Population	
		H_0 true (no difference)	H_a true (difference exists)
Conclusion Based on Sample	Reject H_0	Type I error (α)	Power ($1 - \beta$)
	Fail to reject H_0	Confidence	Type II error (β)

Confidence Intervals

- Reverse of hypothesis testing, start off with a desired confidence (usually 95%) and use a table to find corresponding Z/t values. Scores are then multiplied by standard deviation and then added/subtracted from the mean

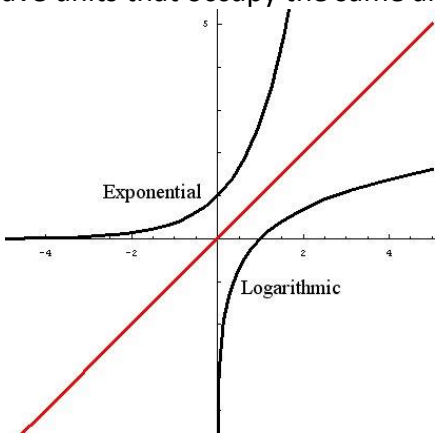
Charts, Graphs, and Tables

Types of Charts

- Pie/Circle Charts: represent relative amounts of entities. Loses impact as number of categories increases.
- Bar Charts and Histograms: Bar charts are used for categorical data, while histograms are for numerical data.
- Box Plots: used to show the range, median, quartiles and outliers for a set of data. **Box-and-whisker** is a labeled box plot.
    - Box: bounded by Q1 and Q3, Q2 is the line in the middle (median).
    - End of Whiskers: largest and smallest values in the data set that are not outliers.
- Maps: data is demonstrated geographically

Graphs and Axes

- Linear Graphs: can be linear, parabolic, exponential or logarithmic
- Axes of a linear graph will have units that occupy the same amount of space



- Semilog and Log-Log Graphs: changes are made to one or both of the **axis ratio's**.

Applying Data

- **Correlation** refers to a connection – direction relationship, inverse relationship, etc. – between data. This does not imply **causation**.